- $val = \mathbb{Z}$;
- $error = \{Err\}$;
- $\triangle\ cxt_f = var \times val + \{\bullet\}$;
- $env = var \to_f val$;
- $\triangle\ env_v = cxt_f \times env$;
- $\triangle\ env_f = name_{proc} \to_f (var \times s)$;
- $out_e = val + error$;
- $out_s = env + error$.

$$
\begin{array}{rcl}
t & ::= & \triangle\ p \\
 & | & e \\
 & | & s \\
 & | & e_x \\
 & | & s_x
\end{array}
$$

$$
\begin{array}{rcl}
e & ::= & c \\
 & | & x \\
 & | & +\ e_1\ e_2
\end{array}
\qquad\qquad
\begin{array}{rcl}
e_x & ::= & +_1\ e_2 \\
 & | & +_2
\end{array}
$$

$$
\begin{array}{rcl}
s & ::= & \mathit{skip} \\
 & | & x := e \\
 & | & s_1; s_2 \\
 & | & \mathit{if}\ e\ s_1\ s_2 \\
 & | & \mathit{while}\ e\ s \\
 & | & \textbf{abort} \\
 & | & \triangle\ f\ (e)
\end{array}
$$

$$
\begin{array}{rcl}
s_x & ::= & x :=_1 \\
 & | & ;_1\ s_2 \\
 & | & \mathit{if}_1\ s_1\ s_2 \\
 & | & \mathit{while}_1\ e\ s \\
 & | & \mathit{while}_2\ e\ s \\
 & | & \triangle\ \mathit{call}_1\ f
\end{array}
$$

$$
\begin{array}{rcl}
\triangle\ p & ::= & s; \\
 & | & f\ (x) := \{s\}\ ; p
\end{array}
$$

$$
\begin{array}{rcl}
st\,(e) & = & \triangle\ env_v \\
st\,(s) & = & \triangle\ env_v \times env_f \\
\triangle\ st\,(p) & = & env_v \times env_f \\
st\,(+_1\ e_2) & = & \triangle\ env_v \times out_e \\
st\,(+_2) & = & val \times out_e
\end{array}
$$

$$
\begin{array}{rcl}
st\,(x :=_1) & = & env \times out_e \\
st\,(;_1\ s_2) & = & \triangle\ cxt_f \times env_f \times out_s \\
st\,(if_1\ s_1\ s_2) & = & \triangle\ env_v \times env_f \times out_e \\
st\,(while_1\ e\ s) & = & \triangle\ cxt_f \times env_f \times out_s \\
st\,(while_2\ e\ s) & = & \triangle\ env_v \times env_f \times out_e \\
\triangle\ st\,(call_1\ f) & = & env_v \times env_f \times out_e
\end{array}
$$

$$
\begin{array}{rcl}
res\,(e_x) & = & out_e \\
res\,(s_x) & = & out_s \\
res\,(e) & = & out_e \\
res\,(s) & = & out_s \\
\triangle\ res\,(p) & = & out_s
\end{array}
$$

$$
\begin{array}{rcl}
abort\,(Err) & = & True \\
abort\,(ret\ E) & = & False \\
\triangle\ abort\,(E_v) & = & False \\
\triangle\ abort\,(E_v, E_f) & = & False \\
abort\,(v, Err) & = & True \\
abort\,(v, val\ v) & = & False \\
abort\,(E, Err) & = & True \\
abort\,(E, val\ v) & = & False \\
\triangle\ abort\,(E_v, Err) & = & True \\
\triangle\ abort\,(E_v, val\ v) & = & False \\
\triangle\ abort\,(C_f, Err) & = & True \\
\triangle\ abort\,(C_f, ret\ E) & = & False \\
\triangle\ abort\,(E_v, E_f, Err) & = & True \\
\triangle\ abort\,(E_v, E_f, val\ v) & = & False
\end{array}
$$

$$
\begin{array}{rcl}
ref & : & cxt_f \to var \to Prop \\
ref((x, v), y) & = & (x = y) \\
ref(\bullet, x) & = & False
\end{array}
$$

$$\frac{\textsc{AbortE}(e_x)}{e_x, \sigma \Downarrow \mathit{Err}} \quad \mathit{abort}\,(\sigma) \qquad \frac{\textsc{AbortS}(s_x)}{s_x, \sigma \Downarrow \mathit{Err}} \quad \mathit{abort}\,(\sigma)$$

$$\frac{\textsc{Abort}}{\mathbf{abort}\,,\ \triangle\ (E_v, E_f) \Downarrow \mathit{Err}} \qquad \frac{\textsc{Cst}(c)}{c,\ \triangle\ E_v \Downarrow \mathit{val}\, c} \qquad \frac{\textsc{VarCxt}(x)\ \triangle}{x, ((y, v), E) \Downarrow \mathit{val}\, v} \quad x = y$$

$$\frac{\textsc{Var}(x)}{x,\ \triangle\ (C, E) \Downarrow \mathit{val}\, v} \quad \frac{E[x] \rightsquigarrow v}{x \in \mathrm{dom}(E)\ \triangle\ \wedge \neg \mathit{ref}(C, x)}$$

$$\frac{\textsc{VarUndef}(x)}{x,\ \triangle\ (C, E) \Downarrow \mathit{Err}} \quad x \notin \mathrm{dom}(E)\ \triangle\ \wedge \neg \mathit{ref}(C, x) \qquad \frac{\textsc{Stat}(s)\ \triangle}{s;, (E_v, E_f) \Downarrow o} \quad s, (E_v, E_f) \Downarrow o$$

$$\frac{\textsc{FunDecl}(f, x, s, p)\ \triangle}{f\,(x) := \{s\}\,; p, (E_v, E_f) \Downarrow o} \quad p, (E_v, E_f\,[f \mapsto (x, s)]) \Downarrow o \qquad \frac{\textsc{FunCall}(f, e)\ \triangle}{f\,(e)\,, (E_v, E_f) \Downarrow o'} \quad e, E_v \Downarrow o \qquad \mathit{call}_1\, f, (E_v, E_f, o) \Downarrow o'$$

$$\frac{\textsc{FunCall1}(f, x, s)\ \triangle}{\mathit{call}_1\, f, ((C, E), E_f, \mathit{val}\, v) \Downarrow o} \quad s, (((x, v), E), E_f) \Downarrow o \quad E_f[f] = (x, s)$$

$$\frac{\textsc{FunCall1Undef}(f)\ \triangle}{\mathit{call}_1\, f, (E_v, E_f, \mathit{val}\, v) \Downarrow \mathit{Err}} \quad f \notin \mathrm{dom}(E_f)$$

$$\textsc{Add}(e_1, e_2)\quad \frac{e_1,\ \triangle\ E_v \Downarrow o \qquad +_1\ e_2, (\ \triangle\ E_v, o) \Downarrow o'}{+\ e_1\ e_2,\ \triangle\ E_v \Downarrow o'} \qquad \textsc{Add}_1\,(e)\quad \frac{e,\ \triangle\ E_v \Downarrow o \qquad +_2, (v_1, o) \Downarrow o'}{+_1\ e, (\ \triangle\ E_v, \mathit{val}\ v_1) \Downarrow o'}$$

$$\textsc{Add}_2\quad \frac{\mathit{add}\ (v_1, v_2) \rightsquigarrow v}{+_2, (v_1, \mathit{val}\ v_2) \Downarrow \mathit{val}\ v} \qquad \textsc{Skip}\quad \frac{}{\mathit{skip},\ \triangle\ ((C, E), E_f) \Downarrow \mathit{ret}\ E}$$

$$\textsc{Asn}(x, e)\quad \frac{e,\ \triangle\ E_v \Downarrow o \qquad x :=_1, (E_v, o) \Downarrow o'}{x := e,\ \triangle\ (E_v, E_f) \Downarrow o'} \qquad \textsc{Asn1Immutable}(x)\quad \frac{\triangle}{x :=_1, ((C, E), \mathit{val}\ v) \Downarrow \mathit{Err}}\ \mathit{ref}(C, x)$$

$$\textsc{Asn}_1\,(x)\quad \frac{E[x \mapsto v] \rightsquigarrow E'}{x :=_1, ((C, E), \mathit{val}\ v) \Downarrow \mathit{ret}\ E'}\ \neg \mathit{ref}(C, x)$$

$$\textsc{Seq}(s_1, s_2)\quad \frac{s_1, ((C, E), E_f) \Downarrow o \qquad ;_1\ s_2,\ \triangle\ (C, E_f, o) \Downarrow o'}{s_1; s_2,\ \triangle\ ((C, E), E_f) \Downarrow o'} \qquad \textsc{Seq}_1\,(s_2)\quad \frac{s_2,\ \triangle\ ((C, E), E_f) \Downarrow o}{;_1\ s_2,\ \triangle\ (C, E_f, \mathit{ret}\ E) \Downarrow o}$$

$$\textsc{If}(e, s_1, s_2)\quad \frac{e,\ \triangle\ E_v \Downarrow o \qquad \mathit{if}_1\ s_1\ s_2,\ \triangle\ (E_v, E_f, o) \Downarrow o'}{\mathit{if}\ e\ s_1\ s_2,\ \triangle\ (E_v, E_f) \Downarrow o'}$$

$$\textsc{If1True}(s_1, s_2)\quad \frac{s_1,\ \triangle\ (E_v, E_f) \Downarrow o}{\mathit{if}_1\ s_1\ s_2,\ \triangle\ (E_v, E_f, \mathit{val}\ v) \Downarrow o}\ v \neq 0$$

$$\textsc{If1False}(s_1, s_2)\quad \frac{s_2,\ \triangle\ (E_v, E_f) \Downarrow o}{\mathit{if}_1\ s_1\ s_2,\ \triangle\ (E_v, E_f, \mathit{val}\ v) \Downarrow o}\ v = 0 \qquad \textsc{While}(e, s)\quad \frac{\mathit{while}_1\ e\ s,\ \triangle\ (C, E_f, \mathit{ret}\ E) \Downarrow o}{\mathit{while}\ e\ s,\ \triangle\ ((C, E), E_f) \Downarrow o}$$

$$\textsc{While1}(e, s)\quad \frac{e, (\ \triangle\ C, E) \Downarrow o \qquad \mathit{while}_2\ e\ s,\ \triangle\ ((C, E), E_f, o) \Downarrow o'}{\mathit{while}_1\ e\ s,\ \triangle\ (C, E_f, \mathit{ret}\ E) \Downarrow o'}$$

$$\textsc{While2True}(e, s)\quad \frac{s,\ \triangle\ ((C, E), E_f) \Downarrow o \qquad \mathit{while}_1\ e\ s,\ \triangle\ (C, E_f, o) \Downarrow o'}{\mathit{while}_2\ e\ s,\ \triangle\ ((C, E), E_f, \mathit{val}\ v) \Downarrow o'}\ v \neq 0$$

$$\textsc{While2False}(e, s)\quad \frac{}{\mathit{while}_2\ e\ s,\ \triangle\ ((C, E), E_f, \mathit{val}\ v) \Downarrow \mathit{ret}\ E}\ v = 0$$